

What is Multiplication?

- joining equal groups together to see how many altogether.
- repeated addition

What is Division?

- splitting / sharing a large group into smaller equal groups.
- repeated subtraction

Multiplication and Division are reverse operations.

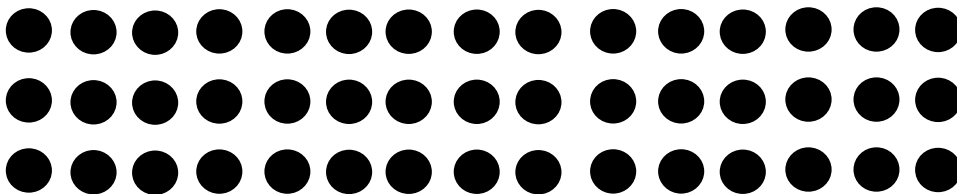
The Language of Multiplication And Division

X	÷
multiply	divide
equal groups of	equal share
times	equal groups
multiples	equal parts
factors	quotient
equal rows of	remainder
array	equal rows of
double, triple	array
product	fraction
	percentage

If Thomas has 15 books and Breanna has 3 times as many books as Thomas, how many books does Breanna have?

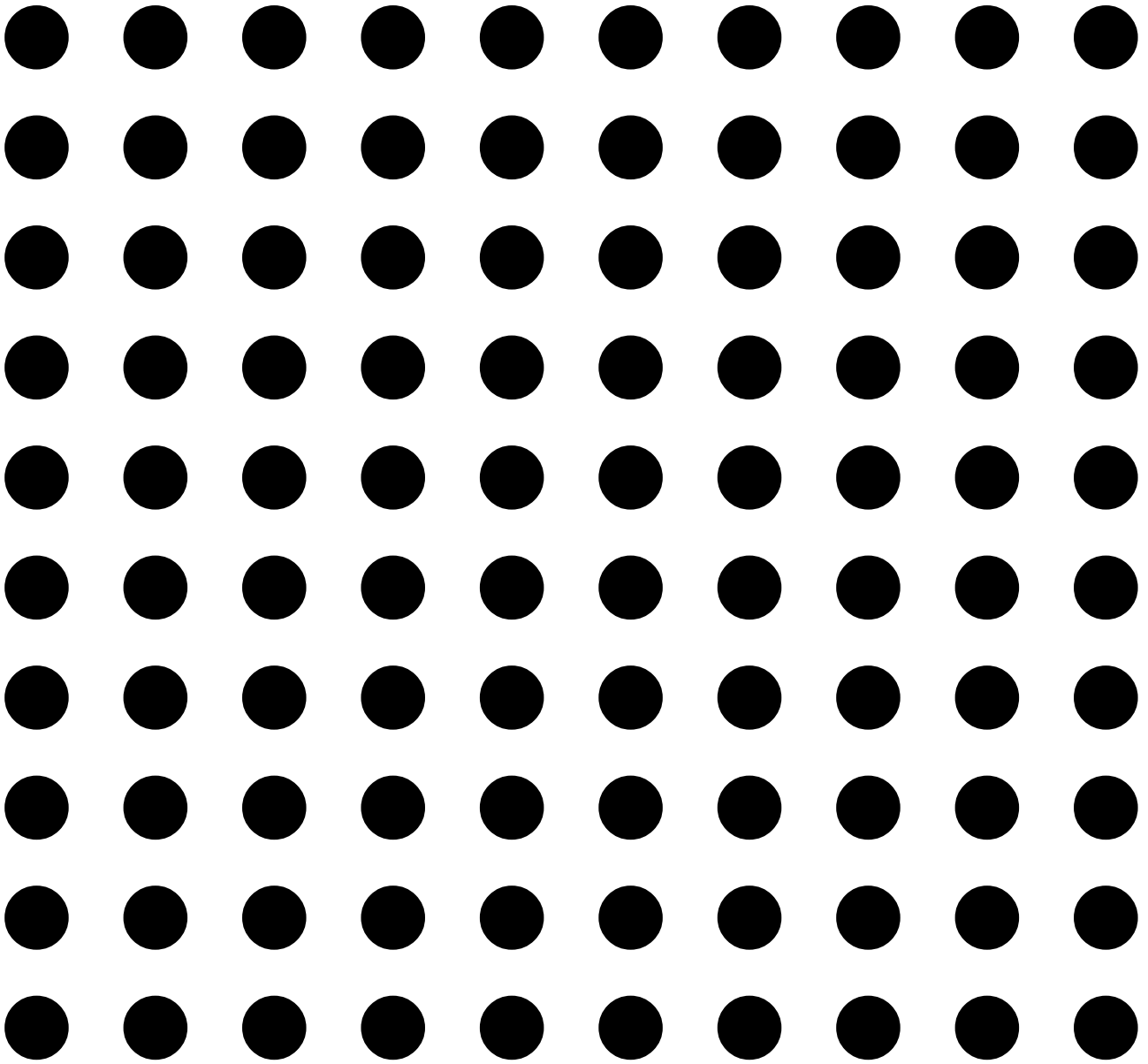
This problem can be solved using multiplication and checked using division.

For example:  $3 \times 15 = 45$  and  $45 \div 3 = 15$



This array can be represented numerically as:  $3 \times 15 = 45$  and  $45 \div 3 = 15$

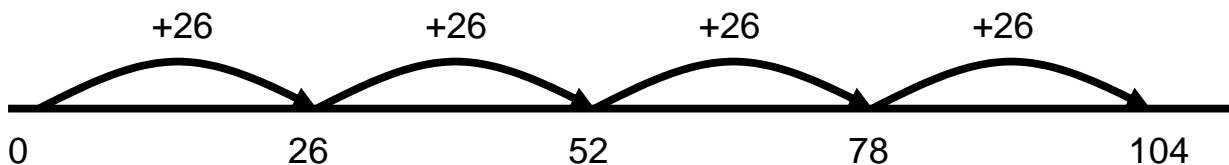
MULTIPLICATION AND DIVISION HANDOUT 4





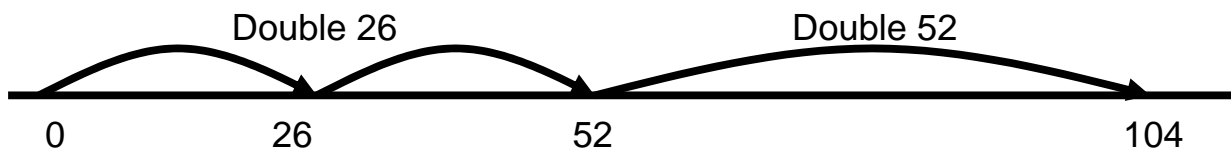
## Mental Strategies for Multiplication and Division

$$26 \times 4 =$$

Repeated Addition

$$26 + 26 = 52, 52 + 26 = 78, 78 + 26 = 104$$

I added 26 and 26 and got 52. Then I added another 26 and got 78. Then I added the 4<sup>th</sup> 26 and got 104.

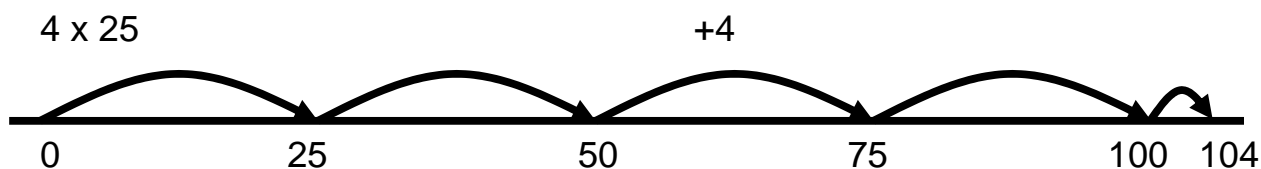
Doubling

$$\text{Double } 26 = 52. \text{ Double } 52 = 104$$

I doubled 26 and got 52. Then I knew I needed another 2 26s which I knew was another 52 so I doubled 52 and got 104.

Rounding and Compensation Strategy

$$4 \times 25$$



$$4 \times 25 = 100, 100 + 4 = 104$$

I knew that 25 times 4 is 100. Then I needed 1 more 4 to make 26 4s. So 100 plus 4 made 104.

Split Strategy

$$4 \times 20 + 4 \times 6$$



$$4 \times 20 = 80, 4 \times 6 = 24 \text{ then } 80 + 24 = 104$$

I knew that 26 was made of 20 plus 6 and that 20 times 4 was 80 and that 6 times 4 was 24.

The Multiplication Algorithm:  
Extended Form

$$\begin{array}{r}
 26 \\
 \times 4 \\
 \hline
 24 \\
 80 \\
 \hline
 104 \\
 \hline
 \end{array}$$

We say:

4 times 6 equals 24, write down the 24. We write a zero in the ones column. Then we say 4 times 2 equals 8, and write it in the tens column to make 80. We then add 24 and 80 to equal 104.

When solving an algorithm, we treat each digit as a 'one', even the 'tens' and 'hundreds'!

A reliance on the algorithm limits children's conceptual understanding of place value.

The Multiplication Algorithm:  
Contracted Form

$$\begin{array}{r}
 ^226 \\
 \times 4 \\
 \hline
 104 \\
 \hline
 \end{array}$$

We say:

4 times 6 equals 24, write down the 4 and carry the 2.

4 times 2 equals 8, plus the 2 equals 10. Write down the 10.

When solving an algorithm, we treat each digit as a 'one', even the 'tens' and 'hundreds'!

A reliance on the algorithm limits children's conceptual understanding of place value.

The Multiplication Algorithm:  
Extended Form - multiplying 2 digits by 2 digits.

$$\begin{array}{r} \overset{1}{2}46 \\ \times 24 \\ \hline \overset{1}{1}84 \\ 920 \\ \hline 1104 \end{array}$$

We say:

4 times 6 equals 24, write down the 4 and carry the 2.

4 times 4 equals 16, plus the 2 equals 18. Write down the 18.

We write a zero in the ones column.

Then we say 2 times 6 equals 12, write down the 2 and carry the 1.

2 times 4 equals 8, plus the 1 equals 9. Write down the 9.

We then add 4 plus 0 equals 4. 8 plus 2 equals 10, write down the 0 and carry the 1. 1 plus 9 equals 10, plus the 1 we carried equals 11.

When solving an algorithm, we treat each digit as a 'one', even the 'tens' and 'hundreds'!

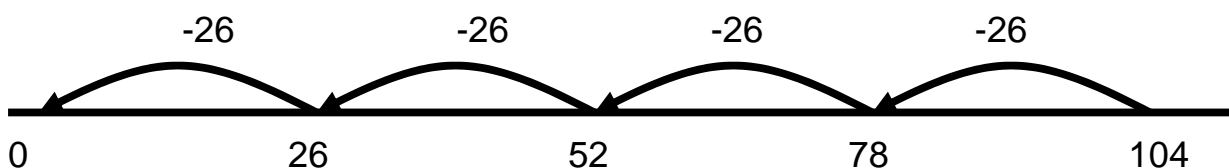
A reliance on the algorithm limits children's conceptual understanding of place value.

## Mental Strategies for Multiplication and Division

$$104 \div 4 =$$

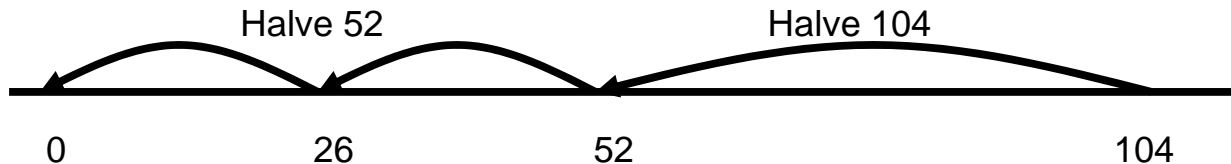
Repeated Subtraction

Repeatedly subtracting 4 from 104 is inefficient – students mix with other strategies such as:



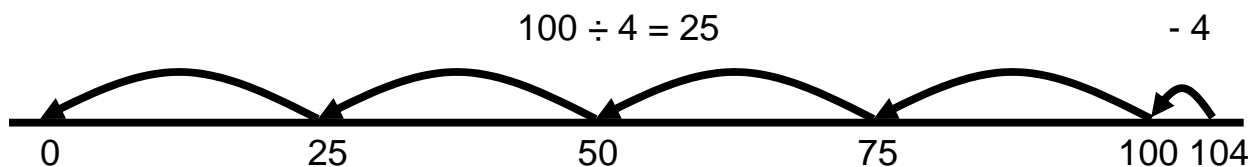
$$104 - 26 = 78, 78 - 26 = 52, 52 - 26 = 26, \\ 26 - 26 = 0.$$

I knew  $25 \times 4$  was 100 so  $26 \times 4$  was going to be 104. I demonstrated on a number line by keeping on subtracting 26.

Halving

$$\text{Halve } 104 = 52. \text{ Halve } 52 = 26$$

I halved 104 and got 52. Then I knew I needed to halve again because dividing by 4 is like finding a quarter and a quarter is half of a half. So I halved 52 and got 26.

Compensation Strategy

$$100 - 4 = 96, 100 \div 4 = 25, 4 \div 4 = 1, 25 + 1 = 26$$

I took 4 away from 104 and got 100. Then did  $100 \div 4$  is 25. I still had the 4 that I took away and  $4 \div 4$  is 1, I added the 1 to 25 and it was 26.

The Division Algorithm:

$$\begin{array}{r} 026 \\ 4 \overline{) 1024} \end{array}$$

We say:

4 into 1 goes 0 times, write down the 0.

4 into 10 goes 2. Write down the 2 above the 10.

$2 \times 4 = 8$  so there are 2 left over, write it in front of the 4.

4 into 24 goes 6, write 6 above the 4.

When solving an algorithm, we treat each digit as a 'one', even the 'tens' and 'hundreds'!

A reliance on the algorithm limits children's conceptual understanding of place value.



### 'Long' Division Algorithm

- 'Long' division follows exactly the same procedure as the division algorithm we just saw.
- 'Long' division is simply a procedure for checking division solutions using multiplication.
- For example, to check if 43 divided by 7 is 6 with 1 remaining, we would multiply 7 by 6 to check.
- As it is a procedure it was often learned by rote with little understanding, resulting in people thinking 'long' division was something very difficult. Children never need to use 'long' division and, like us, many find it confusing.
- The 'long' division algorithm is not in the K-10 Mathematics Syllabus.
- In Stage 4 (Years 7 and 8), students learn to divide two- or three-digit numbers by a two-digit number using strategies such as:
  - to divide a number by 12, first divide by 6 and then double the result
  - to divide a number by 13, first divide the number by ten and then subtract 3 times the number
  - to divide by 20, first halve the number and then divide by 10

$$\begin{array}{r}
 026 \\
 4 \overline{) 104} \\
 \underline{- 8} \phantom{0} \\
 24 \\
 \underline{- 24} \\
 0
 \end{array}$$

4 into 1 goes 0 times, write down the 0

4 into 10 goes 2. Write down the 2.

Check that division fact using multiplication:  $2 \times 4 = 8$ .

Write down the 8 below the 10.

Subtract the 8 to find the remainder:

$10 - 8 = 2$ . Write it below the 8.

Bring down the next number which is 4.

4 into 24 goes 6. Write 6 above the 4.

Check that division fact using multiplication:  $6 \times 4 = 24$ .

Write it below the other 24.

Subtract the 24 to find the remainder:

$24 - 24 = 0$ .

**A Number Activity for Children Working Towards Early Stage One Outcomes**

Pasting Rows

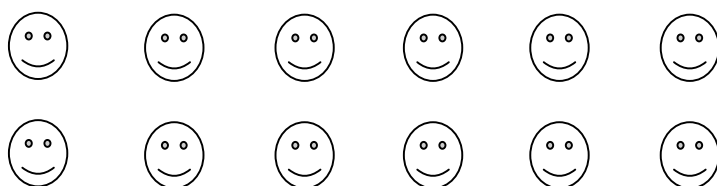
Equipment:

Pictures from magazines, computer etc.

Paper, scissors, glue

Instructions:

The child cuts and pastes pictures or uses a computer drawing program to create arrays. They are asked to describe their array and use numerals / words to label its features.



2	rows of	6
6	rows of	2

2	rows of	6	is	12
6	rows of	2	is	12

Possible questions to ask your child include:

- Is there a different way to make this group?
- What new groups can be made with the same objects?
- How could you check your answer?

Comments when played at home:


### A Number Activity for Children Working Towards Stage One Outcomes

Arrays

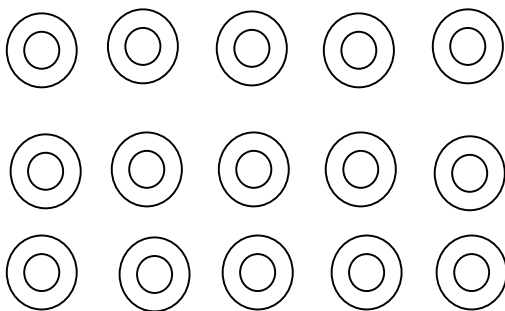
Equipment:

Counters / Beans / Fruit Loops / Buttons

Instructions:

The parent briefly shows a collection of counters arranged as an array on a table.

e.g.



Possible questions include:

- Can you use Counters / Beans / Fruit Loops / Buttons to make what you saw?
- How many Counters / Beans / Fruit Loops / Buttons were there altogether?
- How did you work it out?

Variation: The parent presents a range of Counters / Beans / Fruit Loops / Buttons arranged as arrays. The parent briefly displays one array at a time for the child to determine the total number of counters / beans.

Comments when played at home:


### A Number Activity for Children Working Towards Stage Two Outcomes

#### Factor Chase

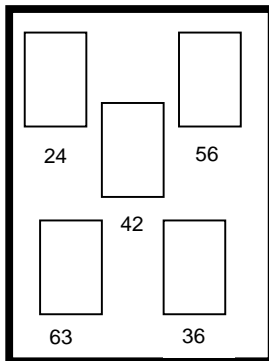
##### Equipment:

Pack of playing cards Ace (=1) through 10.

Paper and pencil

##### Instructions:

Play with your child. Each person flips 10 cards to make 5 pairs. Multiply each pair together and write the product on a piece of paper. Place each pair face down next to its product. For example:



Take turns to suggest factors for each product. For example: Is your 24, 3 x 8? If correct, take the cards and have another turn. If incorrect, think about other factors for the product to suggest on your next turn. The first person to have all of the other person's cards wins.

Comments when played at home:


## A Number Activity for Children Working Towards Stage Two Outcomes

### Factors

#### Equipment:

Pack of playing cards Ace (=1) through 9  
paper and pencil

#### Instructions:

Flip four cards from the top of the deck. Add these cards together. Each player lists all factors of the sum in one minute. For example:

You flipped 7, 8, 3, and 2. When added together, these numbers result in a sum of 24  
( $7 + 8 + 3 + 2 = 20$ ).

Each player lists all factors in one minute.

Players compare factors. Each player gets one point for each correct factor and loses 1 point for each incorrect factor. For example:

You write 2, 10, 4, 5 as the factors of 20.

Your child writes 1, 20, 2, 10, 4, 5 as the factors of 20.

You score 4 points and your child scores 6 points.

The winner is the person with the most points at the end of play.

Comments when played at home:


**A Number Activity for Children Working Towards Stage Three Outcomes**

**Flip 3, make and multiply numbers using mental strategies**

Equipment:

Pencil and paper

Deck of Playing Cards Ace (=1) through 9

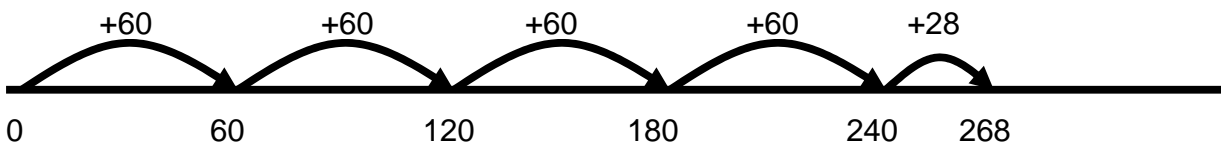
Ace = Represents the number 1

Instructions:

The child is given a pack of cards. The child flips 3 cards. The child is then asked make a two-digit number and a one-digit number. They use mental strategies to multiply the numbers. They record their mental strategies on an empty number line, using numbers and using words. They are encouraged to use a range of mental strategies. For example:

The child flips 4, 6 and 7 and makes  $67 \times 4 =$ .

$60 \times 4 = 240$ ,  $7 \times 4 = 28$ ,  $240 + 28 = 268$ .



First I multiplied the 60 by the 4 and got 240. Then I multiplied the 7 by the 4 and got 28. Then I added the 240 and the 28 and got 268.

Comments when played at home:


**A Number Activity for Children Working Towards Stage Three Outcomes**

Product Chase

Equipment:

Pack of playing cards Ace (=1) through 9  
paper and pencil

Instructions:

Play with your child. Each player takes three cards to make a 2-digit number and a 1-digit number. They multiply the two numbers to find their product. The winner is the person who makes the greater product. For example:

You take 6, 4, and 3. You can make  $64 \times 3$ ,  $63 \times 4$ ,  $43 \times 6$ ,  $34 \times 6$  etc. You choose  $63 \times 4$  because this will give you the greatest product with those numbers.

Each player records their numbers and the strategy they used to calculate the product. The player with the largest product takes all of the cards. The winner is the person who has the most cards when play stops.

Comments when played at home:
